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Recoil proton distribution in high energy photoproduction processes

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Abstract. For high energy linearly polarized photon–proton scattering we have calculated the azimuthal and polar angle distributions on a recoil proton in the inclusive experimental setup. We have taken into account the production of lepton and pseudoscalar meson charged pairs in the point-like approximation. The typical values of the cross sections are of the order of hundreds of picobarn. The size of the polarization effects are of order of several percents. The results are generalized for the case of electroproduction processes on the proton at rest and for a high energy proton production process on a resting proton.

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We have considered the experimental setup of processes of charged pair a_-a_+ production (pseudoscalars, leptons) by high energy photon scattering on a proton in the rest frame followed by the detection of the recoil proton. We used the approximation of point-like particles. The process considered was

$$\gamma(k,\varepsilon)+p(p)\to a_-(q_-)+a_+(q_+)+p(p')\,,$$
$$s=2k.p,\quad k^2=0\,,\quad p^2=(p')^2=M^2,\quad q_-^2=q_+^2=m^2\,. \quad (1)$$

Two different mechanisms of pair production must be considered. One corresponds to Bethe–Heitler (BH) pair creation by two photons. The other one is bremsstrahlung (B), which corresponds to the case when a pair is created by a single virtual photon (we have implied the lowest order contributions in the QED coupling constant $\alpha=1/137$). The contribution of B type is suppressed compared with BH type by a factor of $|q^2|/s$. As to interference of the B and BH amplitudes: it is exactly zero for the recoil proton in the inclusive setup we have considered below.

The accuracy of the formulae given below is determined by the terms we have omitted systematically compared with terms of order of unity,

$$1+O\left(\frac{\alpha}{\pi},\frac{|Q^2|}{s},\frac{m^2}{s},\frac{M^2}{s}\right)\,,\quad Q=p-p'. \quad (2)$$

In the peripheral kinematical region $s\gg|q^2|\sim M^2$, the infinite momentum frame (IMF) or Sudakov [2] parametrization of transferred momentum and 4-momenta of the final particles works effectively:

$$Q=\alpha_q\tilde{p}+\beta_q k+q_\perp,\quad q_\pm=\alpha_\pm\tilde{p}+x_\pm k+q_{\pm\perp}\,,$$
$$c_\perp p=c_\perp k=0,\quad \tilde{p}=p-k\frac{M^2}{s}\,,$$
$$\tilde{p}^2=0,\quad q_\perp^2=-\mathbf{q}^2<0\,. \quad (3)$$

From the on mass shell condition of the recoil proton, $(p-q)^2=M^2$, one infers

$$s\beta_q=-\left(\mathbf{q}^2+M^2\alpha_q\right)/(1-\alpha_q)\approx-\mathbf{q}^2\,. \quad (4)$$

Here we use the smallness of $M^2\alpha_q=(M^2/s)\left(s_1+\mathbf{q}^2\right)$ compared with $\mathbf{q}^2$. Here $s_1=(q_++q_-)^2$ is the invariant mass square of the pair, assumed to be of order $4m^2$.

For the case of large Q one can put the term $Q^2=s\alpha_q\beta_q-\mathbf{q}^2$ equal to $Q^2=-\mathbf{q}^2=-q^2$.

The ratio of transversal and longitudinal components of the momentum of the recoil proton (laboratory frame implied) is

$$\tan\theta=\frac{\mathbf{p}'_\perp}{\mathbf{p}'_{||}}=\frac{|\mathbf{q}|}{(\mathbf{q}^2/2M)}=\frac{2M}{q}\,. \quad (5)$$

This relation, first mentioned in a paper of Benaksas and Morrison [1], can be written in a different form in terms of the value for the 3-vector of the momentum of the recoil proton P,

$$\frac{P}{2M}=\frac{\cos\theta}{\sin^2\theta},\quad q^2=4M^2\cot^2\theta\,, \quad (6)$$

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where θ is the angle between the directions of initial photon and recoil proton in the laboratory frame (see for a more exact formula the appendix).

The matrix element of charged lepton or pion pair production in lowest order of QED perturbation theory (keeping in mind the BH mechanism) has the form

$$M^i = \frac{(4\pi\alpha)^{3/2}}{-q^2} J^p_\nu O^i_{\mu\lambda} \varepsilon^\lambda(k) g^{\mu\nu}, \quad i = l(\text{lept}), \pi(\text{ps}), \tag{7}$$

with the proton current defined as

$$J^p_\nu = \bar{u}(p') \left[F_1(Q^2)\gamma_\nu + \frac{[\hat{Q}, \gamma_\nu]}{4M} F_2(Q^2) \right] u(p)$$

and $F_{1,2}$ is for the form factors of the protons. The Compton lepton tensor has the form

$$O^l_{\mu\lambda} = \bar{u}(q_-) \left[\gamma_\mu \frac{\hat{q}_- - \hat{Q} + m}{D_+} \gamma_\lambda + \gamma_\lambda \frac{\hat{Q} - \hat{q}_+ + m}{D_-} \gamma_\mu \right] v(q_+),$$

and similarly for the Compton pion tensor,

$$O^\pi_{\mu\lambda} = -2g_{\mu\lambda} + \frac{(2q_- - k)_\lambda (Q - 2q_+)_\mu}{D_-} + \frac{(k - 2q_+)_\lambda (2q_- - k)_\mu}{D_+},$$

where $D_\pm = (k - q_\pm)^2 - m^2$. These tensors obey the gauge invariance requirements $O^i_{\mu\lambda} Q^\mu = O^i_{\mu\lambda} k^\lambda = 0$.

Using the Gribov prescription for the Green function of the virtual photon in Feynman gauge and omitting small contributions in the ranges of the declared accuracy,

$$g^{\mu\nu} = g^{\mu\nu}_\perp = \frac{2}{s} [\tilde{p}^\mu k^\nu + \tilde{p}^\nu k^\mu] \approx \frac{2}{s} \tilde{p}^\mu k^\nu,$$

one can write the matrix element, extracting explicitly the factor s, in the form

$$M^i = s \frac{(4\pi\alpha)^{3/2}}{-q^2} N^p \frac{2}{s} \tilde{p}^\mu e^\lambda O^i_{\mu\lambda}, \quad N^p = \frac{1}{s} J^p_\eta k^\eta, \quad i = l, \pi. \tag{8}$$

Both light-cone projections of the proton current and the Compton tensors are finite in the large s limit. Summing over the spin states of the proton one has for the proton current projection square

$$\sum |N^p|^2 = 2F(q^2) = 2 \left[F_1^2(-q^2) + \frac{q^2}{4M^2} F_2^2(-q^2) \right].$$

Expressing the phase volume of the final particles in terms of Sudakov variables [2], we have

$$\mathrm{d}\Gamma = (2\pi)^{-5} \frac{\mathrm{d}^3 q_-}{2\epsilon_-} \frac{\mathrm{d}^3 q_+}{2\epsilon_+} \frac{\mathrm{d}^3 p'}{2E'} \delta^4(p + k - p' - q_- - q_+) = (2\pi)^{-5} \frac{\mathrm{d}^2 q \,\mathrm{d}^2 q_- \,\mathrm{d}x_-}{4s x_- x_+}, \tag{9}$$

where we introduced the unit factor $\mathrm{d}^4 Q \delta^4(p - Q - p')$, and besides this we have used

$$\frac{\mathrm{d}^3 q_-}{2\epsilon_-} = \mathrm{d}^4 q_- \delta(q_-^2 - m^2) = \frac{s}{2} \mathrm{d}^2 q_- \,\mathrm{d}\alpha_- \,\mathrm{d}x_- \delta(s\alpha_- x_- - \mathbf{q}_-^2 - m^2).$$

Further operations, summing over the spin states of the leptons of the square of the matrix element, performing the integration over the energy fractions of the pair x_- and x_+ ($x_- + x_+ = 1$) and its transversal momentum $\mathrm{d}^2\mathbf{q}_-$ (the conservation law implies that $\mathbf{q}_- + \mathbf{q}_+ = \mathbf{q}$), and using the photon polarization matrix $\varepsilon_i \varepsilon_j^* = (1/2)[I + \xi_1\sigma_1 + \xi_3\sigma_3]_{ij}$ ($\xi_{1,3}$ are the Stokes parameters, I is the unit matrix, and σ_i is for the Pauli matrices) is straightforward but tedious. The result can be written in the form

$$\frac{\mathrm{d}\sigma^{\gamma p \to a^i \bar{a}^i p}}{\mathrm{d}\varphi \mathrm{d}\theta} = \frac{1}{2\pi} \frac{\mathrm{d}\sigma_0^{\gamma p \to a^i \bar{a}^i p}}{\mathrm{d}\theta} \left(1 + \Lambda^i(\xi_1 \sin 2\varphi + \xi_3 \cos 2\varphi)\right). \tag{10}$$

The azimuthal angle φ is the angle between two vectors transversal to the photon direction: the linear polarization ε of the photon and $\mathbf{q}$, and we have

$$\frac{\mathrm{d}\sigma_0^{\gamma p \to a^i \bar{a}^i p}}{\mathrm{d}\theta} = \frac{\alpha^3}{3\pi M^2} F(q^2) \frac{\sin\theta}{\cos^3\theta} a^i, \quad a^l = \frac{4L_l}{R_l} + 1 - \frac{m_l^2 L_l}{M^2 R_l} \tan^2\theta, \quad a^\pi = \frac{1}{2}\left(\frac{2L_\pi}{R_\pi} - 1 + \frac{m_\pi^2 L_\pi}{M^2 R_\pi} \tan^2\theta\right). \tag{11}$$

Λ^i is the azimuthal asymmetry

$$\Lambda^i = \frac{b^i}{a^i}, \quad b^l = -\left(1 - \frac{m_l^2 L_l}{M^2 R_l} \tan^2\theta\right), \quad b^\pi = \frac{1}{2}\left(1 - \frac{m_\pi^2 L_\pi}{M^2 R_\pi} \tan^2\theta\right). \tag{12}$$

In (11) and (12) the quantities L_i and R_i are

$$R_i = \sqrt{1 + \frac{m_i^2}{M^2} \tan^2\theta}, \quad L_i = \ln\left(\frac{M}{m_i}\right) + \ln\cot\theta + \ln(1 + R_i).$$

It is interesting to consider the distribution $\mathrm{d}\sigma^{\gamma p \to a^i \bar{a}^i p}/\mathrm{d}q$ of the recoil proton over the value of q. Calculations of this distribution were carried out on the base of the formula that is obtained from (10) and (11) after the substitution

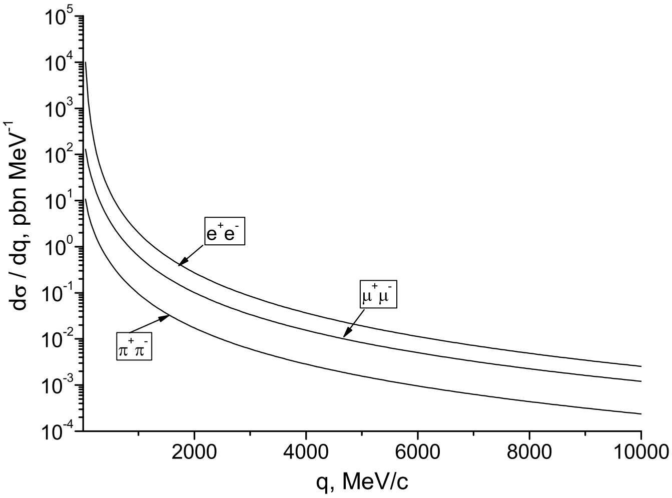

Fig. 1. The distributions $\mathrm{d}\sigma^i/\mathrm{d}q$ in units of $\mathrm{pbn\,MeV}^{-1}$ for the cases of e^+e^- pair, $\mu^+\mu^-$ pair and $\pi^+\pi^-$ pair production as functions of q

$\theta = \arctan(2M/q)$ (see (6)):

$$\frac{\mathrm{d}\sigma^{\gamma p\to a^i\bar{a}^i p}}{\mathrm{d}q} = \frac{8\alpha}{3q^3}F(q^2)\tilde{a}^i\,, \quad i=l,\pi$$
$$\tilde{a}^l = \frac{4q}{\sqrt{4m_l^2+q^2}}\tilde{L}_l + 1 - \frac{4m_l^2}{q\sqrt{4m_l^2+q^2}}\tilde{L}_l\,,$$
$$\tilde{a}^\pi = \frac{1}{2}\left(\frac{2q}{\sqrt{4m_\pi^2+q^2}}\tilde{L}_\pi - 1 + \frac{4m_\pi^2}{q\sqrt{4m_\pi^2+q^2}}\tilde{L}_\pi\right),$$
$$\tilde{L}_i = \ln\left(\frac{q+\sqrt{4m_i^2+q^2}}{2m_i}\right). \tag{13}$$

In Fig. 1 the distributions $\mathrm{d}\sigma^i/\mathrm{d}q$ for each of the processes considered are depicted. For the numerical calculation we used the dipole approximation [3]:

$$F_E = \frac{F_M}{\mu} = \frac{1}{\left(1+\frac{q^2[\mathrm{GeV}^2]}{0.71^2}\right)^2}\,,$$
$$F_E = F_1 - F_2\frac{q^2}{4M^2}, \quad F_M = F_1 + F_2\,.$$

with $\mu = 2.79$ the anomalous magnetic moment of the proton. The function $F(q^2)$ in the dipole approximation has the form

$$F(q^2) = \frac{4M^2+q^2\mu^2}{(4M^2+q^2)\left(\frac{q^2[\mathrm{GeV}^2]}{(0,71)^2}+1\right)^4}\,.$$

In Fig. 2 the asymmetries Λ^i as functions of the momentum q for each of the considered processes are shown.

In Fig. 3 the mentioned asymmetries as functions of the scattering angle θ are shown. The ratio

$$\Lambda_{\mathrm{tot}} = \frac{b^e+b^\mu+b^\pi}{a^e+a^\mu+a^\pi}$$

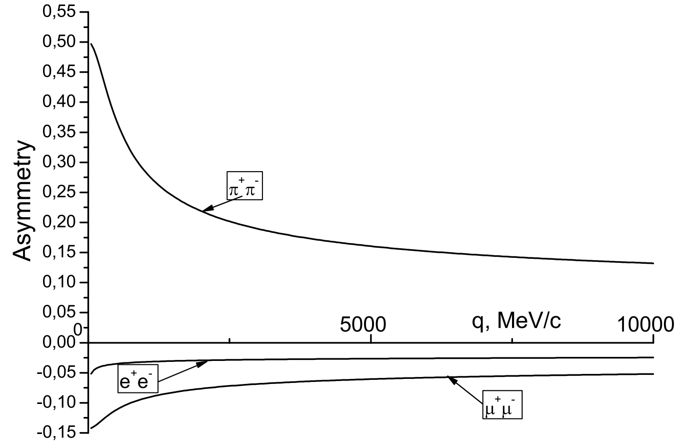

Fig. 2. Asymmetry Λ^i for the cases of e^+e^- pair, $\mu^+\mu^-$ pair and $\pi^+\pi^-$ pair production as a function of q

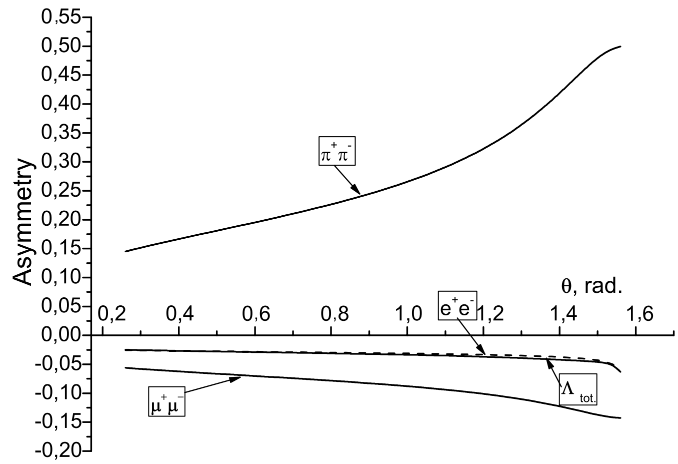

Fig. 3. Asymmetry Λ^i for the cases of e^+e^- pair, $\mu^+\mu^-$ pair and $\pi^+\pi^-$ pair production and also Λ_{tot} as functions of the scattering angle θ

can be considered as the asymmetry averaged over all processes, which estimates the total influence of the linear polarization of the initial photon on the value of the azimuthal asymmetry of the recoil proton. This value is also presented in Fig. 3.

1 Discussion

From Figs. 2 and 3 one can see that in the inclusive setup of the process of charged pair production by interaction of a linearly polarized high energy photon with a proton, the distribution of the recoil proton has a rather essential azimuthal asymmetry, from 0.02 at relatively small polar angles θ up to $\Lambda_{\mathrm{tot}} \sim 0.05$ at $\theta \sim \pi/2$.

In an exclusive setup for the processes with more heavy particles than e^+e^-, the mentioned asymmetry increases. Particularly interesting is the process of $\pi^+\pi^-$ pair photoproduction. One can see that the azimuthal asymmetry of the recoil proton in this process reaches the value $\Lambda^\pi \sim 0.5$ at the region of small transferred momentum or for polar angles close to the value $\theta \sim \pi/2$. The connection of the process of lepton pair production by exclusive photon–proton scattering with generalized parton distributions is investigated in [4].

For the recoil proton in the inclusive setup the distribution is the sum over all possible channels, including fermion (e^+e^-, $\mu^+\mu^-$ and τ^+, τ^-) and pseudoscalar meson ($\pi^+\pi^-$ and K^+K^-) pairs. The production of heavy resonances such as $\rho^\pm$ meson can be excluded using experimental cuts.

The suggested method of measuring the recoil distributions can provide an independent way to control the luminosity and polarization properties of the photon beam.

In [5] the photoproduction of an electron–positron pair on an electron was considered in lowest order of PT. The radiative corrections to the cross section were considered in [6] and in all orders of PT in the parameter $Z\alpha$ in [7] – both for the unpolarized case. It turns out that for $Z < 6$ our results can be applied to the photoproduction on nuclei with a relevant modification of $F(q^2)$. The radiative corrections can change the values of a^i and b^i/a^i in ranges of 1–2%.

The proton recoil momentum measurements can as well be arranged in $ep \to X_{ep'}$ and $pp \to X_{pp'}$ experiments with the initial proton at rest. Using the Weizsäcker–Williams approximation the corresponding cross sections can be written as

$$\mathrm{d}\sigma^{ep\to X_{ep'}} = \frac{2\alpha}{\pi} \int\limits_{2m}^{s/(2M)} \frac{\mathrm{d}\omega}{\omega} \left[\ln \frac{s}{2\omega m_e} - \frac{1}{2}\right] \mathrm{d}\sigma^{\gamma p\to a\bar{a}p'} \tag{14}$$

for electron–proton collisions and

$$\mathrm{d}\sigma^{pp\to X_{pp'}} = \frac{2\alpha}{\pi} \int\limits_{2m}^{s/(2M)} \frac{\mathrm{d}\omega}{\omega} \left[\ln \frac{s}{2M\omega} - \frac{1}{2}\right] \mathrm{d}\sigma^{\gamma p\to a\bar{a}p'}, \tag{15}$$

for proton–proton collisions with $s = 2EM$; E is the energy of the initial electron or proton, and $\mathrm{d}\sigma^{\gamma p\to a\bar{a}p'}$ is given above. Inferring these formulae we supposed that the transversal momentum of projectile (e or p) does not exceed M. The polarization vector of a virtual photon interacting with a proton at rest is directed along the transverse momentum of the projectile.

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Appendix

A more exact formula that takes into account power corrections for the recoil proton momentum has the form [8]

$$p = M \frac{(s-M^2)\left(s-s_1-m^2\right)\cos\theta \pm (s+M^2)\sqrt{D_1}}{4M^2 s + (s-M^2)^2 \sin^2\theta},$$
$$D_1 = \left(s - s_1 + m^2\right)^2 - 4M^2 s - (s-M^2)^2 \sin^2\theta\,. \tag{A.1}$$

Under the condition $s \gg M^2$ the upper branch of (A.1) passes to

$$p = M\left(\frac{2\cos\theta}{\sin^2\theta} - \frac{M^2}{s}\frac{(1+\cos^2\theta)(1+3\cos^2\theta)}{\cos\theta\sin^4\theta} - \frac{(s_1-4m^2)}{s}\frac{(1+\cos^2\theta)}{\cos\theta\sin^2\theta} + O\left(\frac{M^4}{s^2}, \frac{s_1^2}{s^2}\right)\right), \tag{A.2}$$

with s_1 the invariant mass square of the pair produced.

References

1. D. Benaksas, R.J. Morrison, Phys. Rev. **160**, 1245 (1967)
2. V.N. Baier, E.A. Kuraev, V.S. Fadin, V.A. Khoze, Phys. Rep. **78**, 293 (1981).
3. A.I. Akhiezer, M.P. Rekalo, Electrodynamics of Hadrons (Naukova Dumka, Kiev, 1977) (in Russian)
4. E.R. Berger, M. Diehl, B. Pire, Eur. Phys. J. C **23**, 675 (2002)
5. E.A. Vinokurov, E.A. Kuraev, Zh. Eksp. Teor. Fiz. **63**, 1142 (1972)
6. E.A. Vinokurov, E.A. Kuraev, N.P. Merenkov, Zh. Eksp. Teor. Fiz. **66**, 1916 (1974)
7. D. Ivanov, K. Melnikov, Phys. Rev. D **57**, 4025 (1998)
8. V.F. Boldyshev, E.A. Vinokurov, N.P. Merenkov, Y.P. Peresunko, Phys. Part. Nucl. **25**, 292 (1994)